4.9 Antiderivatives

In this section we will introduce the idea of the antiderivative. If we are given the derivative of a function f and we wish to find the original function F, we can take the antiderivative.

Definition: A function F is called an antiderivative of f on an interval I if $F'(x) = f(x)$ for all x in I.

For example, let $f(x) = x^2$. We can use the idea of the power rule to find the antiderivative of f.

Note: If $F(x) = \frac{1}{3}x^3$, then $F'(x) = 3\left(\frac{1}{3}\right)x^2 = x^2 = f(x)$.

We could say that this is our solution but we run into a problem because notice that the derivative of the following functions also equal $f(x)$:

$$G(x) = \frac{1}{3}x^2 + 2$$

$$H(x) = \frac{1}{3}x^2 + 15$$

$$J(x) = \frac{1}{3}x^2 + 2000$$

Notice that we can add any constant to our function and the derivative is still equal to f(x).

All functions in the form of $G(x) = \frac{1}{3}x^2 + C$, where C is a constant, is an antiderivative of f(x).

Theorem: If F is an antiderivative of f on the interval I, then the most general antiderivative of f on I is $F(x) + C$ where C is an arbitrary constant.

The General Antiderivative Formula is: $\frac{x^{n+1}}{n+1} + \mathrm{C}$

Below is a table (found on page 352 of your text) with some of the most commonly used antiderivative that we will encounter. Remember – you must add the constant C at the end of your antiderivative!

Function	Antiderivative		Function	Antiderivative
$cf(x)$	cF(x)		sin (x)	-cos(x)
f(x) + g(x)	F(x) + G(x)		$\sec^2(x)$	tan(x)
x^n (n≠ -1)	$\frac{x^{n+1}}{n+1}$		sec(x)tan(x)	Sec(x)
$\frac{1}{x}$	ln(x)		$\frac{1}{\sqrt{1-x^2}}$	$sin^{-1}(x)$
e^x	e^x		$\frac{1}{1+x^2}$	$tan^{-1}(x)$
b^x	$\frac{b^x}{\ln b}$		Cosh(s)	Sinh(x)
cos (x)	sin (x)		Sinh(x)	Cosh(x)